

# Suppression of error in qubit rotations due to Bloch-Siegert oscillation via the use of off-resonant Raman excitation

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**Abstract:** The Rotation of a quantum bit (qubit) is an important step in quantum computation. The rotation is generally performed using a Rabi oscillation. In a direct two-level qubit system, if the Rabi frequency is comparable to its resonance frequency, the rotating wave approximation (RWA) is not valid, and the Rabi oscillation is accompanied by the so called Bloch-Siegert oscillation (BSO) that occurs at twice the frequency of the driving field. One implication of the BSO is that for a given interaction time and Rabi frequency, the degree of rotation experienced by the qubit depends explicitly on the initial phase of the driving field. If this effect is not controlled, it leads to an apparent fluctuation in the rotation of the qubit. Here we show that when an off-resonant lambda system is used to realize a two-level qubit, the BSO is inherently negligible, thus eliminating this source of potential error.

## 1. Introduction

Quantum computation [1-3] has drawn much attention in recent years. Although the experimental realization of a quantum computer has remained a big challenge, there are many proposals for realizing them physically. A general quantum computer can be realized by performing one and two-gate operations. These operations often require rotation of a qubit. Therefore, precise rotation of a qubit plays an important role in quantum computation. Most qubit rotations are generally performed by using generalized Rabi flopping effect. So far, qubit operations and other elementary model quantum gate operations have been performed in many different physical systems, including nuclear magnetic resonance, ion traps, and Josephson junctions [1-3]. In these systems, the transition frequencies for the qubits are in the range of a few MHz to a few GHz. In general, a fast rotation of a qubit requires a high Rabi frequency, which can easily be on the order of the transition (resonance) frequency, in these systems [1,4,5]

Recently we have shown [6-8] that when a two-level system is resonantly driven by a strong Rabi frequency so that the rotating wave approximation (RWA) is not valid, an effect called the Bloch-Siegert Oscillation (BSO) [9,10] becomes significant compared to the Rabi oscillation. The lowest order of the BSO for a two-level system is due to an effective virtual two-photon transition process. The BSO occurs at twice the frequency of the driving field and its magnitude is proportional to the ratio of the Rabi frequency to the driving frequency. The presence of the BSO modifies the population dynamics of both the ground and the excited states of the two-level system. In particular, the population of either state of the two-level system has a component oscillating at twice the driving frequency. One implication of the BSO is that for a given interaction time and a Rabi frequency, the degree of rotation experienced by a qubit depends explicitly on the *initial phase*

of the driving field. If this effect is not controlled, it may lead to an apparent randomness in the rotation of a qubit.

For a low frequency qubit system that is operated by a fast driving field, the BSO correction to the usual Rabi oscillation could be a significant fraction. For example, in a recent experiment by Martinis *et al.* [11], the BSO amplitude was on the order of 1% of the usual Rabi oscillation. For a still stronger driving field, this amplitude could be as large as 10% of the Rabi amplitude. A quantum computer could be used for real applications only if it has a fault-tolerant capability to a defined degree. It is generally perceived that a bit error rate of the order of  $10^{-6}$  would be necessary for a large scale quantum computer to work properly [12]. Therefore, the introduction of a 1% error through an imprecise rotation of a qubit is unacceptable for most protocols.

To avoid this potential source of error in fast qubit rotation, the Rabi frequency should be much less than the transition frequency. However, this limits the number of operations that can be performed within the decoherence time. One solution may be to keep track of the phase of the field by measuring it locally before each operation, but this may be technically difficult and may make the quantum computation process more complex. In this paper, we show that the effect of the BSO can be avoided without limiting the operating speed if an optically off-resonant Raman excitation is used to produce the Rabi flopping.

The paper is organized as follows: In section II, we briefly review the results of our previous work on direct two-level qubit systems [6,7]. In section III, we illustrate the origin of the BSO in terms of multi-photon transitions using the composite (joint) states of the quantized field and the eigenstates of the atom. In section IV, using the composite state argument, we show that for an effective two-level lambda system where the energy difference between its two lower energy levels is much smaller than the optical frequency, the BSO is inherently negligible. We conclude with a summary in section V.

## 2. General formalism for the population dynamics of a two-level system and the BSO

We consider an ideal two-level system where a ground state  $|0\rangle$  is coupled to a higher energy state  $|1\rangle$ . We also assume that the  $|0\rangle \rightarrow |1\rangle$  transition is magnetic dipolar, with a transition frequency  $\omega$  and the magnetic field is of the form  $B=B_0 \cos(\omega t + \phi)$  [13-19]. The Hamiltonian can be written as,

$$\hat{H} = \varepsilon(\hat{1} - \sigma_z)/2 + g(t)\sigma_x, \quad (1)$$

where  $g(t) = -g_0 [\exp(i\omega t + i\phi) + \text{c.c.}]/2$ ,  $g_0$  is the Rabi frequency,  $\sigma_i (i = x, y, z)$  are the standard Pauli matrices, and  $\varepsilon = \omega$  corresponds to the resonant excitation. We also consider the state vector of the above system to be of the form  $|\xi(t)\rangle = C_0(t)|0\rangle + C_1(t)|1\rangle$ , where  $|0\rangle \equiv [1 \ 0]^\dagger$  and  $|1\rangle \equiv [0 \ 1]^\dagger$  are the matrix representations of the two basis states. After performing rotating wave transformation by a matrix  $\hat{Q} = (\hat{1} + \sigma_z)/2 + \exp(i\omega t + i\phi)(\hat{1} - \sigma_z)/2$ , the Schrödinger equation takes the form (setting  $\hbar=1$ ),

$$\frac{\partial |\tilde{\xi}(t)\rangle}{\partial t} = -i\hat{H}(t)|\tilde{\xi}(t)\rangle, \quad (2)$$

where the effective Hamiltonian in the rotated basis is

$\hat{H} = -g_0[1 + \exp(-i2\omega t - i2\phi)]/2 \cdot \sigma_+ - g_0[1 + \exp(i2\omega t + i2\phi)]/2 \cdot \sigma_-$ , and the state vector in that basis is  $|\tilde{\xi}(t)\rangle \equiv \hat{Q}|\xi(t)\rangle = \tilde{C}_0(t)|0\rangle + \tilde{C}_1(t)|1\rangle$ . Writing this state vector

as  $|\tilde{\xi}(t)\rangle = \sum_{n=-\infty}^{\infty} |\xi_n\rangle \beta^n$ , where  $\beta = \exp(-i2\omega t - i2\phi)$  and  $|\xi_n\rangle \equiv a_n|0\rangle + b_n|1\rangle$ , we get for all  $n$  [6]:

$$\dot{a}_n = i2n\omega a_n + ig_o(b_n + b_{n-1})/2, \quad (3a)$$

$$\dot{b}_n = i2n\omega b_n + ig_o(a_n + a_{n+1})/2. \quad (3b)$$

In Fig. 1, we have shown a pictorial representation of the different terms in the above equation Eq.(3). Here, the coupling between  $a_0$  and  $b_0$  is the conventional one which is present when the RWA is made. The couplings to the nearest neighbors of the terms  $a_0$  and  $b_0$ ,  $b_{\pm 1}$  and  $a_{\pm 1}$ , are detuned by an amount  $2\omega$ , and so on.

Under conditions of adiabatic excitation, we can calculate the population amplitudes of the two levels as follows [6,7,8]:

$$C_0(t) = \text{Cos}(g'_0(t)t/2) - 2\eta\Sigma \cdot \text{Sin}(g'_0(t)t/2), \quad (4a)$$

$$C_1(t) = ie^{-i(\omega t + \phi)}[\text{Sin}(g'_0(t)t/2) + 2\eta\Sigma^* \cdot \text{Cos}(g'_0(t)t/2)], \quad (4b)$$

where we have defined  $g'_0(t) = \frac{1}{t} \int_0^t g_o(t') dt'$ , with  $g_o(t) = g_{0M}[1 - \exp(-t/\tau_{sw})]$  and  $\tau_{sw}$  is the switching time constant,  $\eta \equiv (g_{0M}/4\omega)$  and  $\Sigma \equiv (i/2)\exp[-i(2\omega t + 2\phi)]$ . To lowest order in  $\eta$ , this solution is normalized at all times. Note that if one wants to produce this excitation on an ensemble of atoms using a  $\pi/2$  pulse and measure the population of the excited state  $|1\rangle$  at a time  $t = \tau$  so that  $g'_0(\tau)\tau/2 = \pi/2$ , the result would be a signal given by  $|C_1(g'_0(\tau), \phi)|^2 = (1/2) \times [1 + 2\eta \text{Sin}(2\omega\tau + 2\phi)]$ , which contains information of both the amplitude and the phase of the driving field. This result shows that it is possible to determine both the phase (modulo  $\pi$ ) and

the amplitude of an RF signal coupled to a two-level system by observing the population of one of the levels, as recently demonstrated by us experimentally [8].

A detailed study of the oscillation of the excited state population  $|C_1(\tau)|^2$  as a function of the interaction time  $\tau$  ( i.e., the Rabi-Bloch-Siegert oscillation (RBSO)), have been reported in Ref. [6]. The BSO is the finer rapid oscillation part of the RBSO, and is proportional to  $\eta$  for moderate Rabi frequencies. When the value of the Rabi frequency is increased further, the finer oscillation display higher order harmonics at integer multiples of  $2\omega$ . As such, it is useful to define the generalized BSO (GBSO), which can be identified as the variation in the population of level  $|1\rangle$  from what is expected within the RWA. Figure 2, top panel, shows a plot of the GBSO versus observation time  $\tau$ , under the condition of a nominal  $\pi/2$  excitation. The bottom panel shows the Fourier transform of the GBSO versus  $\tau$  curve. As can be seen, the GBSO amplitude has an oscillation at both  $2\omega$  as well as  $4\omega$ . We have observed both of these harmonics experimentally as well [8].

As discussed in Section I, the GBSO imposes constraints on the precision of rotation of a qubit [6,7]. To illustrate this explicitly, consider a generic situation for an elementary quantum computer/register shown schematically in Fig. 3.

We assume that each qubit is represented by a simple two-level system. The transitions between the states are assumed to be produced by Rabi flopping induced by a microwave field. Consider operations performed on the target qubit as shown by an arrow in the figure. We assume that the energy levels of any qubit are brought to resonance with the driving field by changing its energy levels through a proper scheme (not relevant to our discussion here). Due to the BSO, the effective degree of Rabi flopping depends not only on the amplitude of the field, but also on the phase of the field at the particular spatial point of the target qubit at the moment when the qubit interaction with the microwave field begins. This dependence of the Rabi flopping on the initial phase of the field

potentially complicates the degree of accuracy of rotation of a qubit in a direct two-level system when the driving field is strong.

### 3. A quantum composite state representation of the BSO for a two-level atomic system under direct excitation

We assume that the quantum state of an atom is of the form  $|q\rangle = \sum_{i=0,1} c_i |i\rangle$ , where  $|0\rangle$  and  $|1\rangle$  are the ground and the excited states, respectively. Furthermore, we consider the quantum state of the laser field to be of the form  $|\Phi\rangle = \sum_n c_n |n\rangle$ , where  $|n\rangle$  is a quantized state of  $n$  photons with energy  $n\omega$  ( $\hbar=1$ ), and  $|\Phi\rangle$  is a coherent state. The joint (composite) state of the laser and the two-level atom can be written as

$$|\Psi_{2L}\rangle \equiv |q\rangle \otimes |\Phi\rangle = \sum_i c_i |i\rangle \otimes \sum_n c_n |n\rangle. \quad (5)$$

The atom and field interaction Hamiltonian, *without the RWA*, then can be written as

$$\hat{H}_{I(2L)} = g(\hat{S}_{01} + \hat{S}_{10}) \otimes (a + a^\dagger). \quad (6)$$

Here,  $g$  is the Rabi frequency for the transition  $|0\rangle \rightarrow |1\rangle$  (assumed to be a real quantity);  $a^\dagger$  and  $a$  are the creation and the annihilation operators for the field for a single photon, respectively, of the laser field, and  $\hat{S}_{ij}$  ( $i,j=0,1$ ) is the atomic level transition operator given by  $\hat{S}_{ij} = |i\rangle\langle j|$ .

The transition matrix elements of the above interaction Hamiltonian Eq.(6) can be expressed as:

$$\begin{aligned}
\langle H_{I(2L)} \rangle &= \langle \Psi_{2L} | H_{I(2L)} | \Psi_{2L} \rangle \\
&= g \sum_{j=0,1} c_j^* \sum_{i=0,1} c_i \sum_{n'=-\infty,+\infty} c_{n'}^* \sum_{n=-\infty,+\infty} c_n \langle j | \langle n' | H_{I(2L)} | n \rangle | i \rangle \\
&= g \sum_{j=0,1} c_j^* \sum_{i=0,1} c_i \sum_{n'=-\infty,+\infty} c_{n'}^* \sum_{n=-\infty,+\infty} c_n (\delta_{j,1} \delta_{i,0} + \delta_{j,0} \delta_{i,1}) \cdot (\delta_{n',n-1} \sqrt{n} + \delta_{n',n+1} \sqrt{n+1}) \quad (7)
\end{aligned}$$

The interactions described in the above equation are illustrated in Fig. 4. The joint states of a two-level system in the presence of a laser field and their allowed transitions are shown schematically. The resonance frequency of the two level system ( $|0\rangle$  and  $|1\rangle$ ) is  $\omega$ , and  $g$  is the  $|0\rangle$ - $|1\rangle$  Rabi transition frequency. The joint states, in the left side of the figure, show infinite manifolds of the field with the ground state  $|0\rangle$ , i.e.  $|m\rangle|0\rangle$  ( $m = \dots n-1, n, n+1, \dots$ ), and the subsequent states are separated by an energy  $\omega$ . Similarly, the manifolds of the excited state  $|1\rangle$  with the laser field, i.e.  $|m-1\rangle|1\rangle$  ( $m = \dots n-1, n, n+1, \dots$ ), are shown in the right side of the figure. The states of the same energy are drawn on the same horizontal line, e.g.,  $|n\rangle|0\rangle$  and  $|n-1\rangle|1\rangle$ . The arrows indicate the allowed transitions according to Eq.(7). The transitions indicated by the horizontal arrows (a representative case is illustrated by the ellipse A) are due to one photon joint state transitions, with zero detuning, and the rate of such a transition is the Rabi frequency  $g$ . The inclined arrows (a representative case is illustrated by the ellipse B) indicate transitions that are detuned by a frequency  $2\omega$  and the rate of such a transition is also at frequency  $g$ . These transitions are responsible for the lowest order BSO. It is instructive to compare this diagram (Fig.4) with the one shown in Fig.1. The A and B transitions here correspond to the A and B transitions in that diagram. Other allowed transitions which are detuned by  $4\omega$ ,  $6\omega$ , *etc.* are also possible; however, the amplitudes for these processes are increasingly weaker. The GBSO consist of all the off resonant transitions, i.e. transitions at  $2\omega$ ,  $4\omega$ , *etc.* .



#### 4. A Raman transition in an effective 2-level Lambda system and the absence of BSO

In a Raman system, states  $|0\rangle$  and  $|2\rangle$  are the two low-lying nearby states, and a third level  $|1\rangle$  differs by an optical frequency from both of these states. Now, we apply two off-resonant driving fields at frequencies  $\omega_{01} + \delta$  for the  $|0\rangle \leftrightarrow |1\rangle$  transition and  $\omega_{12} + \delta$  for the  $|1\rangle \leftrightarrow |2\rangle$  transition, where  $\delta$  is the detuning. This is illustrated in Fig. 5.(a). For simplicity, we consider that the optical Rabi frequencies for the interactions  $|0\rangle \leftrightarrow |1\rangle$  and  $|1\rangle \leftrightarrow |2\rangle$  are the same and of a magnitude  $g$ . The difference frequency is defined as  $\Delta\omega = \omega_{21} - \omega_{01}$ . These assumptions result in an effective Rabi oscillation (Raman-Rabi oscillation) between the levels  $|0\rangle$  and  $|2\rangle$  at the effective Raman Rabi frequency of magnitude  $\Omega_{02} \approx g^2 / 4\delta$  for  $\delta \gg g$ . In the context of quantum computing, the states  $|0\rangle$  and  $|2\rangle$  would correspond to the quantum states of a qubit.

It is tempting to conclude that this effective two-level lambda system behaves exactly the same way as an actual two-level system where a field at the frequency  $\Delta\omega$  is applied to couple states  $|0\rangle$  and  $|2\rangle$ . In particular, one might think that when the RWA is not made, this effective two level system will also experience a BSO with a magnitude of the order of  $\Omega_{02} / 4\omega$ . However, this is not the case, as we show below.

The 3-level interaction Hamiltonian, without the RWA, can be written as the sum of the interaction of the atom with the two different field modes as follows:

$$\begin{aligned} \hat{H}_{I(3L)} &= \hat{H}_{I_1(3L)} + \hat{H}_{I_2(3L)} \\ &= [g (S_{01} + S_{10}) \otimes (a_1 + a_1^\dagger)] + [g (S_{12} + S_{21}) \otimes (a_2 + a_2^\dagger)]. \end{aligned} \quad (8)$$

Here,  $g$  is the Rabi frequency for each of the two transitions,  $|0\rangle \rightarrow |1\rangle$  and  $|1\rangle \rightarrow |2\rangle$ ;  $a_1^+$  and  $a_1$  are the creation and the annihilation operators, respectively, of a photon by the laser field mode 1 which excites the  $|0\rangle \rightarrow |1\rangle$  transition, and  $a_2^+$  and  $a_2$  are the creation and annihilation operators, respectively, for the laser field mode 2 which excites the  $|1\rangle \rightarrow |2\rangle$  transition.  $S_{ij}$  ( $i, j=0,1,2$ ) is the atomic level projection operator given by  $S_{ij} = |i\rangle\langle j|$ .

The joint (composite) state of the 1 and 2 laser modes with the 3-level atom can be written as

$$|\Psi_{3L}\rangle \equiv \left[ \sum_{i=0,1,2} c_i |i\rangle \otimes \sum_{m=-\infty, +\infty} c_m |m\rangle \otimes \sum_{n=-\infty, +\infty} c_n |n\rangle \right]. \quad (9)$$

The transition matrix elements due to the atom-field interaction, without the RWA, can be obtained from Eqs. (8) and (9) :

$$\begin{aligned} \langle H_{I(3L)} \rangle &= \langle \Psi_{3L} | H_{I(3L)} | \Psi_{3L} \rangle \\ &= g \sum_{j=0,1,2} c_j^* \sum_{i=0,1,2} c_i \sum_{m'=-\infty, +\infty} c_{m'}^* \sum_{m=-\infty, +\infty} c_m \sum_{n'=-\infty, +\infty} c_{n'}^* \sum_{n=-\infty, +\infty} c_n \langle j | \langle m' | \langle n' | H_{I_1(3L)} | n \rangle | m \rangle | i \rangle \\ &\quad + g \sum_{j=0,1,2} c_j^* \sum_{i=0,1,2} c_i \sum_{m'=-\infty, +\infty} c_{m'}^* \sum_{m=-\infty, +\infty} c_m \sum_{n'=-\infty, +\infty} c_{n'}^* \sum_{n=-\infty, +\infty} c_n \langle j | \langle m' | \langle n' | H_{I_2(3L)} | n \rangle | m \rangle | i \rangle, \\ &= g \sum_{j=0,1,2} c_j^* \sum_{i=0,1,2} c_i \sum_{m'=-\infty, +\infty} c_{m'}^* \sum_{n=-\infty, +\infty} c_n (\delta_{j,1} \delta_{i,0} + \delta_{j,0} \delta_{i,1}) \times \\ &\quad \times (\delta_{n',n-1} \sqrt{n} + \delta_{n',n+1} \sqrt{n+1}) \cdot \delta_{m',m} \end{aligned}$$

$$\begin{aligned}
& + g \sum_{j=0,1,2} c_j^* \sum_{i=0,1,2} c_i \sum_{m'=-\infty,+\infty} c_{m'}^* \sum_{m=-\infty,+\infty} c_m (\delta_{j,2} \delta_{i,1} + \delta_{j,1} \delta_{i,2}) \times \\
& \times (\delta_{m',m-1} \sqrt{m} + \delta_{m',m+1} \sqrt{m+1}) \delta_{n',n}
\end{aligned} \tag{10}$$

The composite state picture of the Raman interactions, according to Eq.(10), is illustrated in Fig. 5(b). For simplicity and without any effect on our basic objective, we assume  $\omega_{01} \approx \omega_{21} \equiv \omega$  in what follows. This figure shows the joint states of a 3-level atom with two modes of the laser field and their allowed transitions. Here  $g$  is the Rabi frequency for each transition ( $|0\rangle \rightarrow |1\rangle$  and  $|1\rangle \rightarrow |2\rangle$ ). The number state for mode 1(2) is represented by  $|n\rangle(|m\rangle)$ . We further assume that  $\delta \ll \omega$  and negligible in this diagram. The energy levels of atom plus fields composite states,  $|j\rangle|k\rangle|i\rangle$ , ( where  $j = \dots, m-1, m, m+1, \dots$ ,  $k = \dots, n-1, n, n+1, \dots$  and,  $i = 0, 1, 2$  ) are shown in three columns for  $|0\rangle$ ,  $|1\rangle$ , and  $|2\rangle$  states. Composite states that have the same energy are shown on the same horizontal line. The allowed transitions are given according to Eq. (10). The horizontal arrows represent the resonant one-photon transitions responsible for the usual Rabi flopping with no detuning. The inclined arrows represent the non-resonant two-photon transitions detuned by a frequency  $2\omega$ , and are responsible for the BSO effect on each of the optical transitions ( $|0\rangle \rightarrow |1\rangle$  and  $|2\rangle \rightarrow |1\rangle$ ). The transition path shown by the shading A is the resonant type transition from  $|0\rangle \rightarrow |2\rangle$  via  $|1\rangle$ , without BSO. The transition path shown by shading B represents the lowest order BSO type transition from  $|0\rangle \rightarrow |2\rangle$ . The transition path shown by shading C represents the next order BSO. Note that there is no direct BSO effect for the transition  $|0\rangle \rightarrow |2\rangle$ , but only via the state  $|1\rangle$ . Thus the BSO effect for any  $|0\rangle \rightarrow |2\rangle$  transition will be associated with *at least one* 2-photon transition detuned by a frequency  $2\omega$ . Therefore, the BSO effect for  $|0\rangle \rightarrow |2\rangle$  Raman-Rabi oscillation is of the order of  $g/\omega$ , and *not* of the order of  $\Omega_{02}/\Delta\omega$ . While  $\Omega_{02}/\Delta\omega$  can be

very large, and easily exceed unity, the value of  $g/\omega$  is very small and negligible for all practical implementation of an off-resonant Raman excitation involving optical intermediate states. For example, if  $\omega \sim 10^{15} \times 2\pi \text{ sec}^{-1}$  and  $g$  is  $10^6 \times 2\pi \text{ sec}^{-1}$ , then the value of  $g/\omega \sim 10^{-9}$ . This makes the BSO amplitude negligible in an effectively two-level lambda system.

## 5. Conclusions

We have analyzed the origin of the Bloch-Siegert shift and the Bloch-Siegert oscillation caused by strong driving fields. For a direct two-level qubit system, the final state of a qubit rotation is a function of the absolute local phase of the driving field in the presence of the Bloch-Siegert oscillation. This introduces potential errors in the rotation of such a qubit. We have also analyzed the case in which a qubit is realized for an effectively two-level lambda system, where the qubit rotation is performed via an optically off-resonant Raman transition. In this case we have shown that the BSO is negligible, even when the effective Rabi frequency for the qubit rotation exceeds the transition frequency. We conclude that qubits formed by a Raman excited microwave transition (lambda system) may be more controllably rotated than direct microwave two-level qubits. Therefore, the qubits which are made of lambda systems may be better suited than the qubits that are made of direct two level systems for quantum computation and quantum information processing.

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### Figure captions

Figure 1. Schematic illustration of the multiple order interactions when the rotating wave approximation is not made. The strength of the first-order interaction (shown by circle B), for example, is weaker than the zeroth-order interaction (shown by circle A) by the ratio of the Rabi frequency  $g_0$ , and the effective detuning  $2\omega$ . When the RWA is made, the effective terms are  $a_0$  and  $b_0$ .

Figure 2. (Top) The generalized BSO (GBSO) plotted as a function of the observation time, with the degree of excitation kept constant corresponding to a nominal  $\pi/2$  excitation under the RWA. The GBSO amplitudes are visible at the frequencies  $2\omega$  and  $4\omega$  because of a larger Rabi frequency. (Bottom) The Fourier transform of the amplitude of the GBSO (top figure) shows the presence of two prominent frequencies at  $2\omega$  and  $4\omega$ , where  $\omega$  is the frequency of the driving field.

Figure 3. The schematic picture shows spatially fixed qubits (spins) and their energy levels inside a quantum computer. A microwave spatial field amplitude that interacts with the target qubit is shown by a sinusoidal curve varying in space (at a fixed time). The BSO effect makes the transition probability depend not only on the amplitude, but also on the phase of the microwave field at the spatial position of the target qubit.

Figure 4. Illustration of the joint states of a two level system in the presence of a laser field and their allowed transitions. The resonance frequency of the two level system ( $|0\rangle$  and  $|1\rangle$ ) is  $\omega$ , and  $g$  is the  $|0\rangle$ - $|1\rangle$  Rabi transition frequency. The joint states, in the left side of the figure, show infinite manifolds of the field with the ground state  $|0\rangle$ , i.e.  $|k\rangle|0\rangle$

( $k = \dots, n-1, n, n+1, \dots$ ), and the subsequent states are separated by an energy  $\omega$ . Similarly, the manifolds of the excited state  $|1\rangle$  with the laser field, i.e.  $|k-1\rangle|1\rangle$  ( $k = \dots, n-1, n, n+1, \dots$ ), are shown in the right side of the figure. The states of the same energy are drawn on the same horizontal line, e.g.,  $|n\rangle|0\rangle$  and  $|n-1\rangle|1\rangle$ . The arrows indicate the allowed transitions according to Eq. (7). The transitions indicated by the horizontal arrows (a representative shown by ellipse A) are due to one photon joint state transitions, with zero detuning, and the rate of each transition is the Rabi frequency  $g$ . The inclined arrows (a representative case is illustrated by the ellipse B) indicate transitions that are detuned by a frequency  $2\omega$  and the rate of such a transition is also at frequency  $g$ . These transitions are responsible for the lowest order BSO.

Figure 5.(a). Schematic picture of a Raman transition in an effective 2-level Lambda system. (b). Composite state picture of the Raman interactions, according to Eq.(10), is illustrated. For simplicity and without any loss of generality, we assume  $\omega_{01} \approx \omega_{21} \equiv \omega$ . The figure shows the joint states of a 3-level atom with two modes of the laser field and their allowed transitions. Here  $g$  is the Rabi frequency for each transition ( $|0\rangle \rightarrow |1\rangle$  and  $|2\rangle \rightarrow |1\rangle$ ). The number state for mode 1(2) is represented by  $|n\rangle$  ( $|m\rangle$ ). We further assume that  $\delta \ll \omega$  and negligible in this diagram. The energy levels of atom plus fields composite states,  $|j\rangle|k\rangle|i\rangle$ , (where  $j = \dots, m-1, m, m+1, \dots$ ,  $k = \dots, n-1, n, n+1, \dots$  and,  $i = 0, 1, 2$ ) are shown in three columns for  $|0\rangle$ ,  $|1\rangle$ , and  $|2\rangle$  states. Composite states that have the same energy are shown on the same horizontal line. The allowed transitions are given according to Eq. (10). The horizontal arrows represent the resonant one-photon transitions responsible for the usual Rabi flopping with no detuning. The inclined arrows represent the non-resonant two-photon transitions detuned by a frequency  $2\omega$ , and are responsible



for the BSO effect on each of the optical transitions ( $|0\rangle \rightarrow |1\rangle$  and  $|2\rangle \rightarrow |1\rangle$ ). The shading A show the resonance transition path from  $|0\rangle$  to  $|2\rangle$  via  $|1\rangle$ . The transition shown by shading B represents the lowest order BSO type transition from  $|0\rangle \rightarrow |2\rangle$ . The shading C represents the next order BSO type transition. As can be seen in the figure, there is no direct BSO effect for the transition  $|0\rangle \rightarrow |2\rangle$ , but only via the state  $|1\rangle$ . Thus the BSO effect for any  $|0\rangle \rightarrow |2\rangle$  transition will be associated with *at least one* 2-photon transition detuned by a frequency  $2\omega$ . Therefore, the BSO effect for  $|0\rangle \rightarrow |2\rangle$  Raman-Rabi oscillation is *not* of the order of  $\Omega_{02} / \Delta\omega$  but of the order of  $g/\omega$  which can be practically negligible.

# Figures

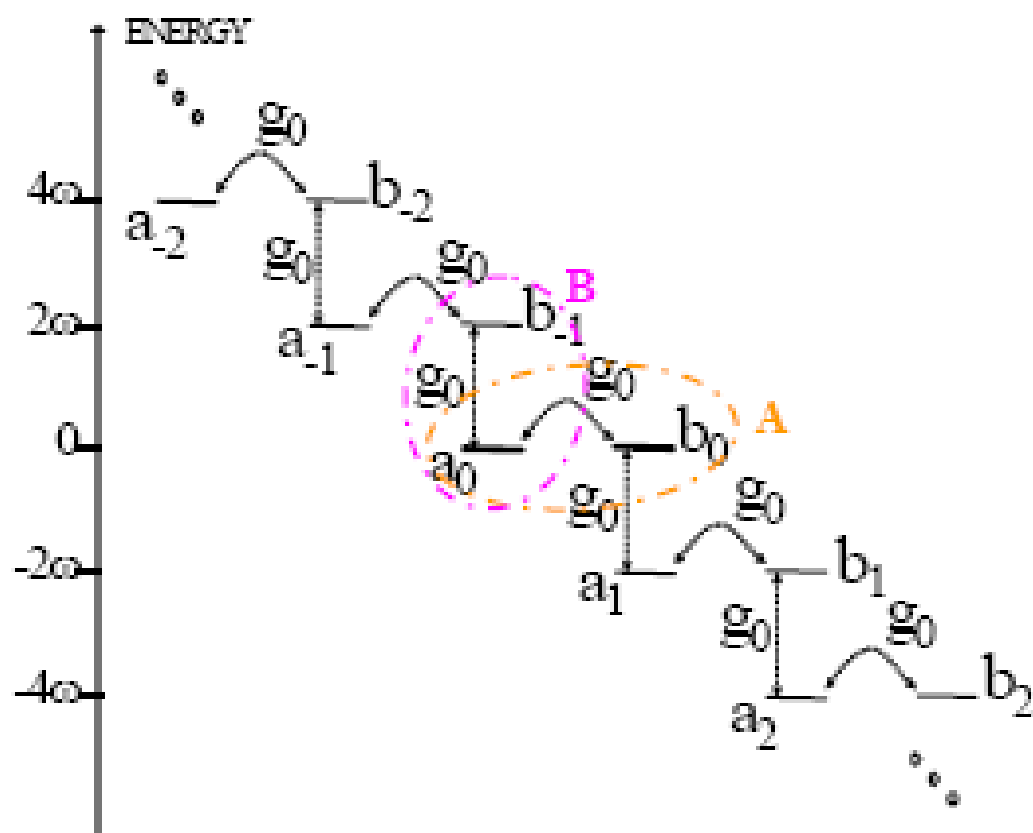
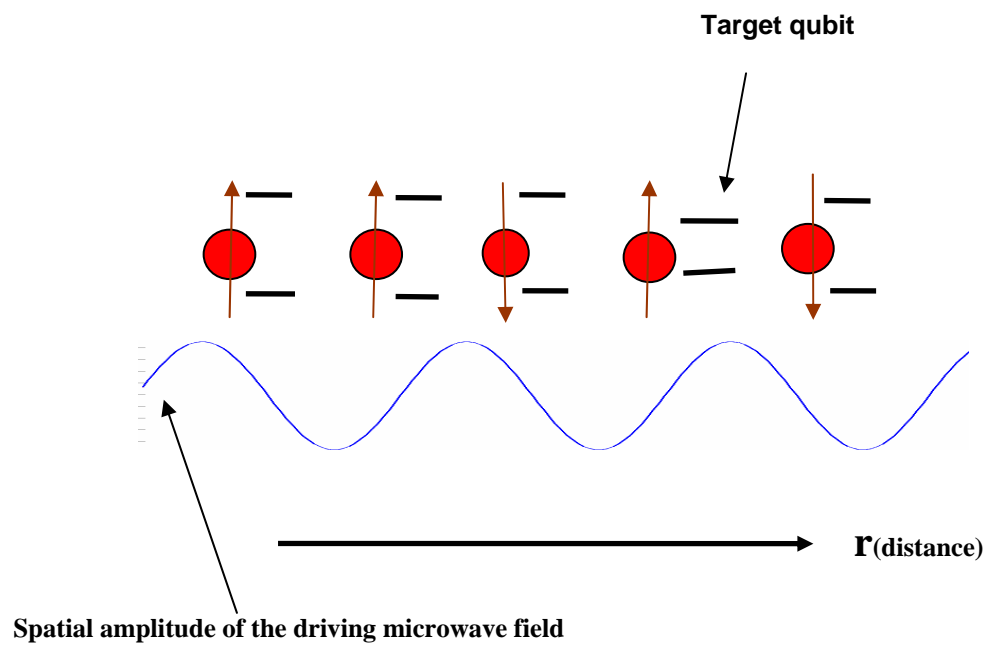
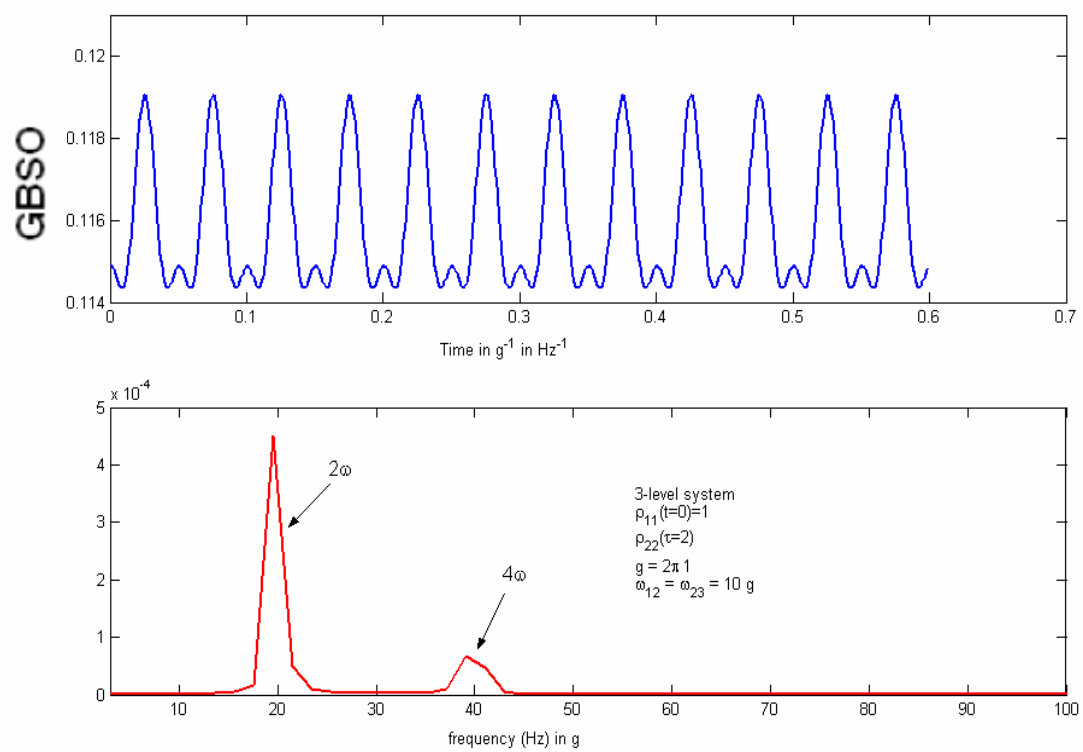


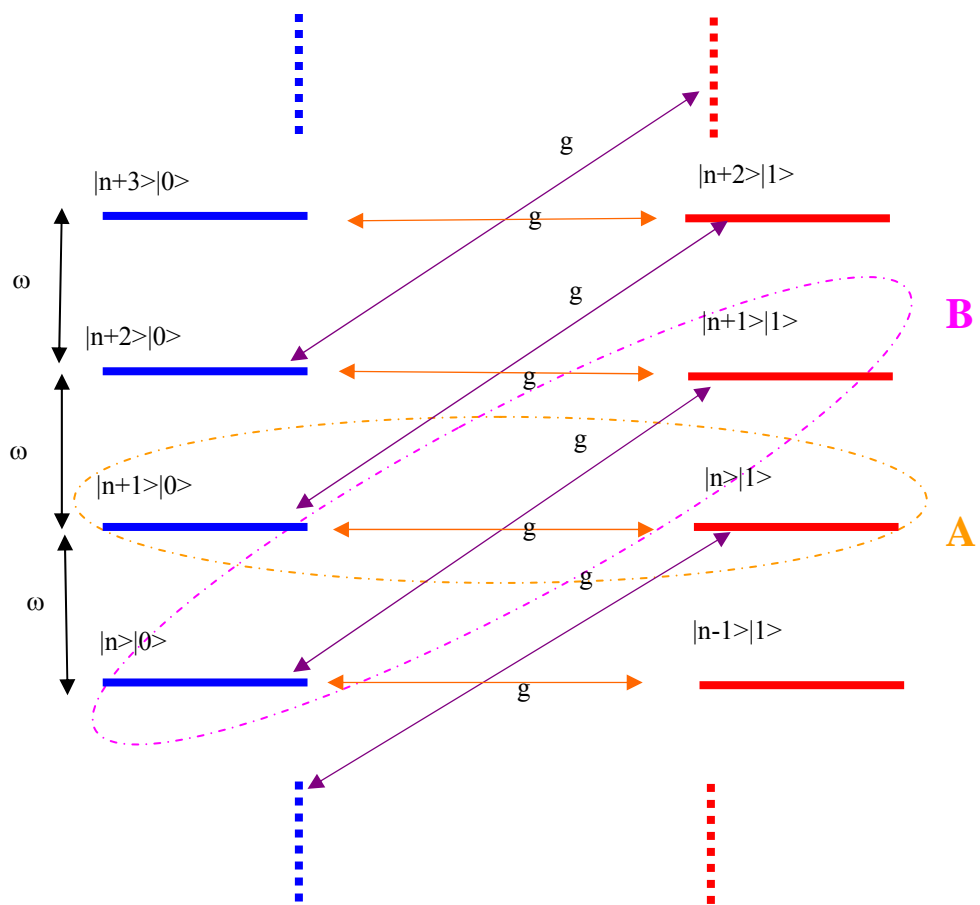
Figure 1



**Figure 2**



**Figure 3**



**Figure 4**

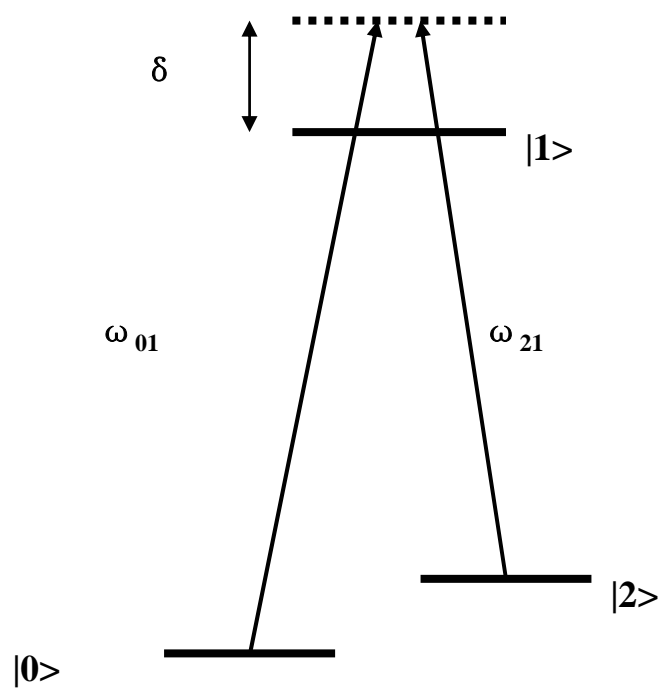


Figure 5(a)

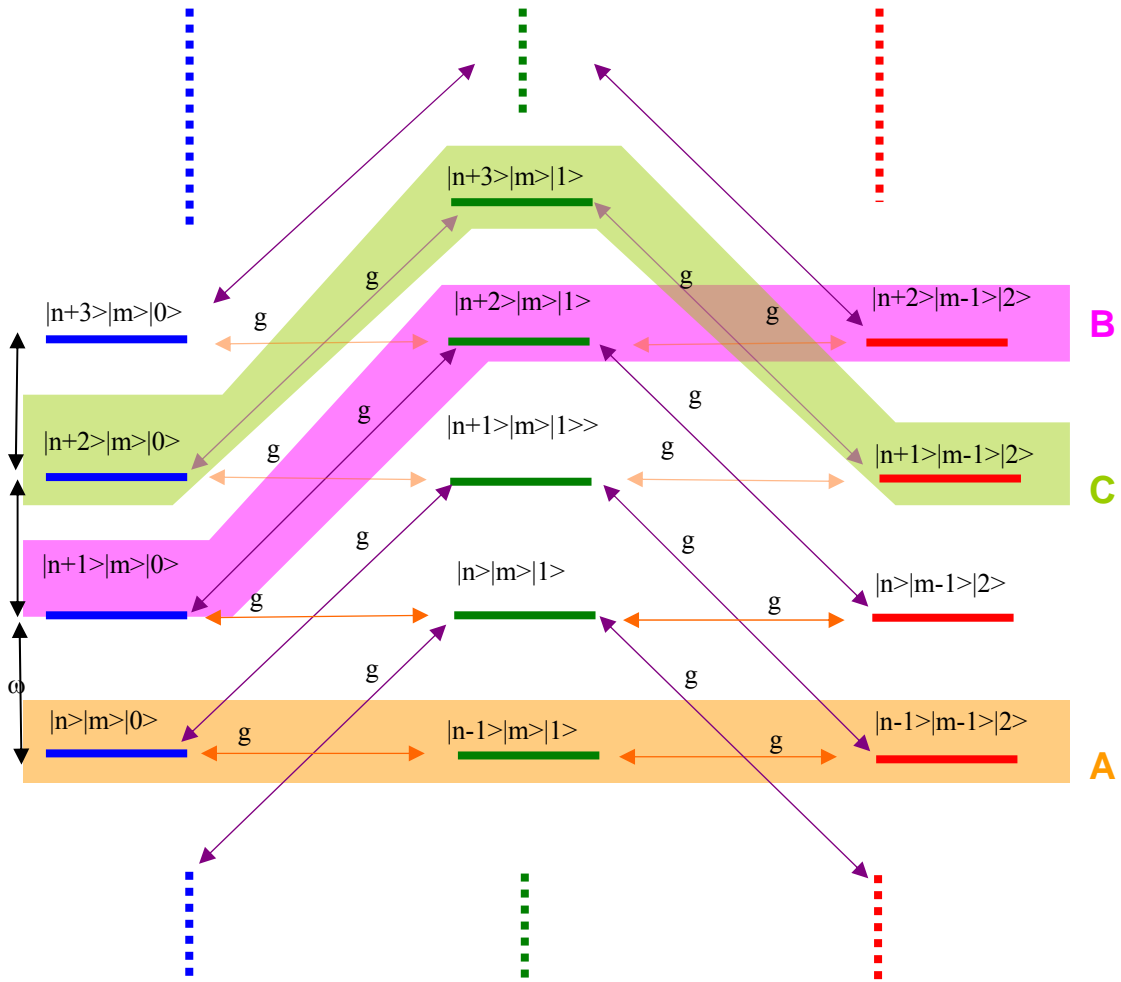


Figure 5(b)